Analytical model for axial stress-strain behavior of welded reinforcement grid confined concrete columns

Tavio* and B. Kusuma

(Received: February 16, 2015; Accepted: August 13, 2015; Published online: September 30, 2015)

Abstract: In a previous study by the authors, the innovative technology of applying welded reinforcement grids (WRG) as confining steel of reinforced concrete columns (instead of traditional ties) under compression loading was experimentally investigated. A stress-strain model of concrete which takes confinement effects into account is developed based on the results of compression loading tests of reinforced concrete column specimens. The stress-strain model is formulated by evaluating the relationship between several key parameters (effective confining pressure, peak stress, strain, and ductility) and the stress-strain behavior observed in the experiments. Analytical results are then compared to experimental values available in the literature. It is shown that the predicted stress-strain relation provides better agreement with the experimental results than existing models.

Keywords: confined concrete, lateral reinforcement, reinforced concrete column, stress-strain model, welded reinforcement grids (WRG).

1. Introduction

Column confinement is an important component of the seismic design of reinforced concrete columns [1-5]. The confinement of core concrete improves the overall strength and stability of the structure subjected to large seismic lateral forces [6-12]. The characteristics of confined concrete have been researched extensively, and the primary parameters of confinement have been identified both experimentally and analytically [13-19]. Various studies on the confinement effects of lateral reinforcement in column have already been conducted [20-24].

It is now well recognized that both the strength and ductility of concrete compressive members can be greatly enhanced using welded reinforcement grids (WRG) [25-29]. Very few experimental and analytical studies are available for compressed members confined by WRG. WRG can be flexibly prefabricated to meet the required size and volumetric ratio of transverse steel. The volumetric ratio is defined as the volume of the confinement steel with respect to the volume of column, obtained by multiplying the gross cross-sectional area and the spacing of confining elements. These grids when used as column transverse reinforcement, could potentially lead to savings associated with easy and fast cage assembly, and reduction in steel consumption since the overlapping reinforcement, bends and bend extensions are eliminated. Furthermore, the precision of welded grid corners, as compared to bent conventional hoop corners, provides better and consistent support to longitudinal reinforcement, also improving column behavior. The grid corners may also provide additional confinement pressure peaks without any longitudinal reinforcement, improving the uniformity of pressure.

The objectives of this study are as follows: 1) to study the applicability of existing confinement models, and 2) to propose a stress-strain model of confined concrete which is applicable to a wider range of WRG ratios than the existing models.

2. Assessment of existing stress-strain confinement models

Several analytical stress-strain relations for rectilinearly confined concrete have been proposed. Among these, the relations presented by Hoshikuma et al. [30] and Legeron and Paultré [31] will be studied to determine the applicability to the specimens. Details of Hoshikuma’s and Legeron’s
Table 1 – Details of specimens tested by Tavio et al. [33]

<table>
<thead>
<tr>
<th>Column specimen ID</th>
<th>Grid configuration</th>
<th>Transverse reinforcement</th>
<th>Longitudinal reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Dia. (mm)</td>
<td>s (mm)</td>
</tr>
<tr>
<td>N1-30W2-4.8</td>
<td>2 x 2</td>
<td>30</td>
<td>4.81</td>
</tr>
<tr>
<td>N1-45W2-3.2</td>
<td></td>
<td>45</td>
<td>3.21</td>
</tr>
<tr>
<td>N1-60W2-2.4</td>
<td></td>
<td>60</td>
<td>2.40</td>
</tr>
<tr>
<td>N1-72W2-2.0</td>
<td></td>
<td>72</td>
<td>2.00</td>
</tr>
<tr>
<td>N1-90W2-1.6</td>
<td></td>
<td>90</td>
<td>1.60</td>
</tr>
<tr>
<td>N1-120W2-1.2</td>
<td></td>
<td>120</td>
<td>1.20</td>
</tr>
<tr>
<td>N2-30W2-4.8</td>
<td></td>
<td>30</td>
<td>4.81</td>
</tr>
<tr>
<td>N2-45W2-3.2</td>
<td></td>
<td>45</td>
<td>3.21</td>
</tr>
<tr>
<td>N2-72W2-2.0</td>
<td></td>
<td>72</td>
<td>2.00</td>
</tr>
<tr>
<td>N1-30W3-4.8</td>
<td>3 x 3</td>
<td>30</td>
<td>4.79</td>
</tr>
<tr>
<td>N1-45W3-3.2</td>
<td></td>
<td>45</td>
<td>3.19</td>
</tr>
<tr>
<td>N1-60W3-2.4</td>
<td></td>
<td>60</td>
<td>2.39</td>
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<td>1.99</td>
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<td>90</td>
<td>1.60</td>
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<td>N1-120W3-1.2</td>
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<td>1.20</td>
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<td>N2-30W3-4.8</td>
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<td>4.79</td>
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<tr>
<td>N2-45W3-3.2</td>
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<td>3.13</td>
</tr>
<tr>
<td>N2-72W3-2.0</td>
<td></td>
<td>72</td>
<td>1.99</td>
</tr>
</tbody>
</table>

confinement models can be found in the literature [30,31] and will not be given here.

Hoshikuma’s model is applicable to normal-strength concrete. It incorporates all the relevant parameters of confinement, including the type, volumetric ratio, and spacing of transverse reinforcement as well as section geometry. The developed model can be used for concrete confined by different types of section geometry including circular, square, and wall-type sections. It can also be used for bridge columns which have larger concrete sections and, thus, lower volumetric ratios of transverse reinforcement.

Legeron’s model is applicable for normal- and high-strength concrete columns based on the large number of test results of circular, square, and rectangular columns tested under various research studies that included the studies undertaken by themselves and a number of other researchers. The strengths ranged from 20 to 140 MPa and the tie yield strengths ranged from 300 to 1400 MPa. The model incorporates almost all the parameters of confinement. The stress-strain relationship was basically same as proposed by Cusson and Paultre [32], but the parameters of the model were recalibrated on the basis of large number of test data collected by the authors.

3. Modeling of stress-strain relation

The stress-strain model has been proposed based on test data by Tavio et al. [33]. This model is applicable to columns with square sections made of normal strength concrete and welded reinforcement grids (WRG). In this section, the equations used in the stress-strain model are modified by performing regression analysis on all test results of the specimens listed in a paper reported by Tavio et al. [33] except for columns laterally reinforced with conventional ties. Table 1 summarizes the details of specimens tested by Tavio et al. [33]. The cross sections of the tested specimens can be seen in Fig. 1. The perspective view of the application of WRG
as transverse reinforcement in RC column is shown in Fig. 2.

The mathematical expressions of the ascending part were originally proposed by Popovics [34], and later used by Cusson and Paultre [32] for high strength concrete. They are expressed by Eq. (1). In this research, all parameters \( r, x, f'_{cc} \) in Eq. (1) are modified based on experimental results as follows:

\[
 f'_{c} = \frac{f'_{cc} x}{r - 1 + x'}
\]  

(1)

where \( r = E_{ce} / (E_{ce} - f'_c) \); \( f'_c \) is the compressive strength of confined concrete; \( e'_{cc} \) is the axial strain at peak strength of confined concrete; \( x = e'_c / e_{cc} \) is the normalized strain; \( f_e \) is the concrete stress corresponding to strain \( e_e \) and \( E_e \) is the modulus of elasticity of unconfined concrete.

The following expression, originally proposed by Carrasquillo et al. [35] is found to produce good agreement with experimentally obtained values, where \( f'_c \) is in Mega Pascal.

\[
 E_c = 3320 f'_c + 6900
\]  

(2)

The linear portion of the descending curve, extends to the level of the stress drop after maximum stress to be 20 percent of the peak stress and is based on the expression proposed by Martinez et al. [36] as described below.

\[
 f_e = f'_{cc} \left[ 1 - 0.15 \left( \frac{e_e - e_{cc}}{e_{cc85} - e_{cc}} \right) \right]
\]  

(3)

Eq. (3), represented by a straight line, passes the strain corresponding to 85 percent of the maximum stress of the descending part, \( e_{cc85} \), for the confined concrete. The proposed stress-strain relationship consists of a nonlinear ascending branch up to confined peak stress and a linear descending branch beyond the peak, as illustrated in Fig. 3.

3.1 Effective confining pressure

In this study, the effective confining pressure, \( P_e \), is defined according to Eq. (4) [21,22].

\[
 P_e = \frac{d_i \left( 1 - \frac{s}{b} \right) \rho_s f_y}{c_i}
\]  

(4)
where $d$, and $c$, are the wire size and the length of one grid cell of welded reinforcement grids (WRG), respectively; $s$ is the longitudinal spacing of transverse reinforcement; $b$ is the shortest dimension of the column section; $\rho$, and $f_{yk}$ are the volumetric ratio and yield strength of welded reinforcement grids (WRG), respectively.

### 3.2 Peak stress, strain, and ductility

A regression analysis was performed on all test results to formulate the peak strength, $f'_{cc}$, the strain at peak strength, $\varepsilon_{cc}$, and the strain corresponding to 85 percent of the peak strength of the descending branch, $\varepsilon_{cc85}$. Figure 4 shows the relationship between the enhancement strength of confined concrete and the effective confinement index, $P_s / f'_{co}$, whereas Figure 5 shows the relationship between the strain gain and the effective confinement index, $P_s / f'_{co}$. 

The results of regression analysis are as follows:

$$f'_{cc} = f'_{co} \left[ 1 + 3.89 \left( \frac{P_s}{f'_{co}} \right)^{0.49} \right]$$  

(5)

where $f'_{co}$ is the compressive strength of unconfined concrete.

$$\varepsilon_{cc} = \varepsilon_{co} \left[ 1.31 \text{EXP} \left( 24.63 \left( \frac{P_s}{f'_{co}} \right) \right) \right]$$  

(6)

Figure 6 shows the relationship between the ductility gain and the effective confinement index. The ductility gain is defined as the difference between the strains at which the stress drops to 85 percent of the peak strength of confined concrete, $\varepsilon_{cc85}$, and of unconfined concrete, $\varepsilon_{co}$. Based on the results of regression analysis shown in Fig. 5, the expression for $\varepsilon_{cc85}$ can be derived as

$$\varepsilon_{cc85} = \varepsilon_{co} + 0.13 \left( \frac{P_s}{f'_{co}} \right)^{0.7}$$

(7)

where the peak strain of the unconfined concrete, $\varepsilon_{co}$, is expressed as per the recommendation of Foster and Gilbert [37].

$$\varepsilon_{co} = 0.002 + 0.001 \left( \frac{f'_{cc} - 20}{80} \right)$$

(8)

The in-place strength of unconfined concrete, $f'_{cc}$, was taken to be 0.85 $f'_{cc}$ for all columns. Similar values were used for columns tested by others [26,27].
4. Comparison of predictions and experimental results

The comparisons between the analytical confined concrete strength values obtained from the proposed confinement model and the experimental strength values are presented in Fig. 7. Strong correlation between the analytical strength and the experimental strength is apparent. For the peak strain $\varepsilon_{cc}$ and strain at 85 percent of the peak strength, agreements between the measured and analytical results are also quite good as shown in Figs. 8 and 9.

Comparisons were made between the predictions of the models and the experimental results of reinforced concrete columns confined with WRG. The study carried out by Holland [26] and Hong [27] included the column specimens with volumetric ratio ranged from 1.0 to 4.0 percent as given in Table 2. The comparisons of the model to the experimental results of all eighteen confined specimens are shown in Figs. 10-13. Curves obtained from the models of Hoshikuma et al. [30] and Legeron and Paultre [31] are also shown in each figure. In all cases, the stress-strain relation proposed by this research accurately reproduces the experiment result. It could be stated that the proposed model generally provides better agreement with the stress-strain relation of confined concrete over a wider range of welded reinforcement grids (WRG) ratios than previous conventional confinement models. The inaccurate predictions of both Legeron and Paultre and Hoshikuma et al. models on the peak stress and strain ductility are mainly due to the derivations of these models from the experimental results of traditionally-confined columns. Hence, the models are not applicable to WRG-confined columns.

Table 2 – Details of specimens tested by others [26, 27]

<table>
<thead>
<tr>
<th>Reference</th>
<th>Label</th>
<th>Grid configuration</th>
<th>$f'_{cu}$, MPa (ksi)</th>
<th>$d_r$, mm (in.)</th>
<th>$s$, mm (in.)</th>
<th>$\rho_t$, %</th>
<th>$f_{th}$, MPa (ksi)</th>
<th>Grid arrangement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holland</td>
<td>W-1</td>
<td>2 × 2</td>
<td>41.8 (6.1)</td>
<td>3.0 (0.118)</td>
<td>25.4 (1.00)</td>
<td>1.36</td>
<td>373 (54.0)</td>
<td><img src="image" alt="Grid Arrangement" /></td>
</tr>
<tr>
<td>[26]</td>
<td>W-2</td>
<td></td>
<td>39.3 (5.7)</td>
<td>4.1 (0.161)</td>
<td>19.1 (0.75)</td>
<td>3.13</td>
<td>576 (83.5)</td>
<td><img src="image" alt="Grid Arrangement" /></td>
</tr>
<tr>
<td>W-3</td>
<td>3 × 3</td>
<td></td>
<td>43.4 (6.3)</td>
<td>2.4 (0.094)</td>
<td>31.8 (1.25)</td>
<td>0.99</td>
<td>436 (63.2)</td>
<td><img src="image" alt="Grid Arrangement" /></td>
</tr>
<tr>
<td>W-4</td>
<td>3 × 3</td>
<td></td>
<td>38.5 (5.6)</td>
<td>2.4 (0.094)</td>
<td>12.7 (0.50)</td>
<td>2.38</td>
<td>436 (63.2)</td>
<td><img src="image" alt="Grid Arrangement" /></td>
</tr>
<tr>
<td>W-5</td>
<td>4 × 4</td>
<td></td>
<td>37.3 (5.4)</td>
<td>2.0 (0.079)</td>
<td>25.4 (1.00)</td>
<td>1.00</td>
<td>288 (41.7)</td>
<td><img src="image" alt="Grid Arrangement" /></td>
</tr>
<tr>
<td>Hong</td>
<td>W-6</td>
<td>2 × 2</td>
<td>47.6 (6.9)</td>
<td>4.1 (0.161)</td>
<td>19.1 (0.75)</td>
<td>4.05</td>
<td>496 (71.9)</td>
<td><img src="image" alt="Grid Arrangement" /></td>
</tr>
<tr>
<td>[27]</td>
<td>W-7</td>
<td>3 × 3</td>
<td>47.6 (6.9)</td>
<td>2.5 (0.098)</td>
<td>19.1 (0.75)</td>
<td>1.79</td>
<td>436 (63.2)</td>
<td><img src="image" alt="Grid Arrangement" /></td>
</tr>
<tr>
<td>W-8</td>
<td>3 × 3</td>
<td></td>
<td>47.6 (6.9)</td>
<td>2.0 (0.079)</td>
<td>25.4 (1.00)</td>
<td>1.23</td>
<td>288 (41.7)</td>
<td><img src="image" alt="Grid Arrangement" /></td>
</tr>
<tr>
<td>W-9</td>
<td>4 × 4</td>
<td></td>
<td>47.6 (6.9)</td>
<td>2.7 (0.106)</td>
<td>25.4 (1.00)</td>
<td>2.15</td>
<td>397 (57.5)</td>
<td><img src="image" alt="Grid Arrangement" /></td>
</tr>
<tr>
<td>W-10</td>
<td>4 × 4</td>
<td></td>
<td>47.6 (6.9)</td>
<td>2.7 (0.106)</td>
<td>19.1 (0.75)</td>
<td>2.87</td>
<td>398 (57.5)</td>
<td><img src="image" alt="Grid Arrangement" /></td>
</tr>
</tbody>
</table>
For verification of the proposed model, Figures 14 and 15 compare the corresponding experimental and predicted stress-strain curves of a few representative test specimens only listed in Table 2. It may be seen from these figures that the agreement between the predictions from the proposed model and the experimental stress-strain curves is very satisfactory.

Figures 10-15 indicate that the proposed model predicts the experimental results more accurately than the other models, especially along the descending part of the stress-strain curve. This is because such previous models were developed for traditional tied and/or spiral columns, which are not as ductile as columns transversely reinforced with WRG. It can also be concluded that the proposed model can predict the actual stress-strain curves.
Fig. 11 – Comparison between experimental and theoretical stress-strain curves of confined concrete for $2 \times 2$ WRG with eight longitudinal bars

Fig. 12 – Comparison between experimental and theoretical stress-strain curves of confined concrete for $3 \times 3$ WRG with four longitudinal bars

Fig. 13 – Comparison between experimental and theoretical stress-strain curves of confined concrete for $3 \times 3$ WRG with twelve longitudinal bars
more accurately for a wide range of experimental results [33] than the other models, particularly in the descending branch of the curves.

5. Conclusions

Based on the experimental results from eighteen RC column specimens tested by Tavio et al. [33], the confinement effects of welded reinforcement grids (WRG) on the stress-strain behavior of normal- and high-strength concrete were investigated and a stress-strain relationship model is proposed. The following concluding remarks can be drawn:

(1) When the same amounts of welded reinforcement grids (WRG) were used, the gains in the strength and ductility of normal strength concrete confined by $3 \times 3$ welded reinforcement grids (WRG) (9-cells) were predicted higher by the proposed model than those of concrete confined by $2 \times 2$ welded reinforcement grids (WRG) (4-cells) only.

(2) The ductility of columns is shown to be dependent on confinement index, $\rho_s f_{y, s}/f'$. It is concluded that the strain ductility ratio increases with an increase in $\rho_s f_{y, s}/f'$ for a given welded reinforcement grids (WRG).

(3) The proposed model can predict the stress-strain curves more accurately for a wide range of experimental results [33] than the other models, particularly in the descending branch of the curves. The proposed model has also been verified with other experimental data [26, 27]. It also provides a simple computational procedure without requiring any iterative cal-
culation.

Acknowledgements

The authors would like to express their sincere gratitude to all the research project members on Innovative Development of WRG for Seismic-Resistant Structures. Their generous contributions are deeply appreciated. All opinions, findings, conclusions, and recommendations in the paper are those of the authors.

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